

Theoretical Necessity of an External Scalar Field in the Kaluza-Klein Theory (I)

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Abstract

We show that the principle of least action is generally inconsistent with the usual Kaluza-Klein program, the higher dimensional Einstein-Hilbert action being unbounded from below. This inconsistency is also present in other theories with higher dimensions like supergravity. Hence, we conclude to the necessity of an external scalar field to compensate this flaw.

1 Introduction

Landau and Lifshitz [1] have shown that the principle of least action is meaningful for Einstein general relativity only if the gravitation constant, G , is positive. First they subject the metric tensor, $(g_{\mu\nu})_{\mu,\nu=0,1,2,3}$, to four (the number of spacetime coordinates, x^μ ; $x^0 = ct$) conditions which suppress the liberty of a gauge choice. With these restrictions, the variations $\delta g_{\mu\nu}$ correspond really to a change of the gravitational field and not to a coordinates transformation. They make the choice

$$g_{0k} = 0 \quad \text{and} \quad \det(g_{kl}) = \text{constant}, \quad (1)$$

where $k, l = 1, 2, 3$. The only relevant terms of the Einstein-Hilbert action, $-\frac{1}{2\chi} \sqrt{-g} R$, involving time derivatives of the metric tensor are thus of the form (the signature of the metric will be + - - - and $g = \det(g_{\mu\nu})$ throughout this paper) :

$$\frac{1}{8\chi} g^{00} g^{kl} g^{mn} \frac{\partial g_{km}}{\partial x^0} \frac{\partial g_{ln}}{\partial x^0} \sqrt{-g} \quad (2)$$

which reduce to

$$\frac{1}{8\chi} g^{00} \left(\frac{\partial g_{kl}}{\partial x^0} \right)^2 \sqrt{-g} \quad (3)$$

in local spatial cartesian coordinates ($g_{kl} = g^{kl} = \delta_{kl}$). Now, since $g^{00} = 1/g_{00} > 0$ ($g_{\mu\nu}g^{\mu\nu} = 4$ and $g_{kl}g^{kl} = 3$), the sign of the quantity (3) above is evident and well defined. If the Einstein gravitational constant $\chi = 8\pi G/c^4$ were negative, sufficiently rapid changes of the g_{kl} with respect to time would lead to arbitrarily large negative values of the action and thus to instabilities without limit. Hence, Landau and Lifshitz conclude that G should be positive. Indeed, as emphasized by Noerdlinger [2], it would not be possible to replace the principle of least action with a principle of greatest action since physical examples, such as the free motion of a test particle, satisfy the principle of least action¹.

In the Brans-Dicke theory [3, 4], a scalar field, ϕ , enters in the metric and the action may be expressed in two possible frames conformally related one to the other: the Jordan-Fierz frame and the Einstein-Pauli frame². Noerdlinger [2] has shown that a similar argument applies in the Jordan-Fierz frame to the additive term

$$\frac{\omega}{\phi} \left(\frac{\partial \phi}{\partial x^0} \right)^2 \sqrt{-g}, \quad (4)$$

which similarly requires the positivity of the Brans-Dicke coupling constant³, ω , with the implicit assumption of a positive defined scalar field. Let us notice that the positivity of ϕ (see Hawking and Ellis [5]) also follows from the argument of Landau and Lifshitz, though not emphasized by Noerdlinger. Moreover, the argument of

¹Nevertheless, let us recall that the requirement of an extremal action is sufficient to derive the field equations.

²In the Jordan-Fierz frame the action terms for the ordinary matter (other than the Brans-Dicke scalar and non-gravitational) take the general relativistic form whereas in the Einstein-Pauli frame the gravitational term of the action is of the Einstein-Hilbert form.

³This was first put forward by Brans and Dicke from physical considerations concerning the positivity of the contribution to the inertial reaction (*i.e.* to the Brans-Dicke scalar) from nearby matter [3].

Landau and Lifshitz holds both in Einstein-Pauli frame and Jordan-Fierz frame, independently of their respective physical significance. Thus, since the constraint on the allowed values of ω implied by the argument of Landau and Lifshitz is stronger in Jordan-Fierz frame ($\omega > 0$) than in Einstein-Pauli frame ($2\omega + 3 > 0$), clearly it is the former that should be considered as relevant for our purpose. These results seem largely unknown⁴. Indeed, even today, one often finds in the literature some studies on the Brans-Dicke theory with $\omega < 0$ although the variational principle is postulated. In particular, the low-energy effective action of string theory, in the graviton-dilaton sector, can be given in the form of an effective Brans-Dicke action with coupling constant $\omega = -1$ which, according to the previous argument, would lead to devastating instabilities. This feature of string theory has raised no discussion in the literature hitherto. Let us emphasize that the argument of Landau and Lifshitz, based on the requirement of a lower bound for the action, is stronger than the usual argument based on the weak energy condition. Quite often [8, 9], an argument based on the weak energy condition is invoked which leads to the weaker constraint $2\omega + 3 > 0$. This constraint is compatible with the low-energy string limit $\omega = -1$.

2 The case of the Kaluza-Klein theory

Here we apply an analogous to Landau and Lifshitz argument to Kaluza-Klein theory. First, let us consider the case of a classical real scalar field, ϕ , minimally coupled to gravity. As one knows, the action of the system writes

$$S = \int \sqrt{-g} \left(-\frac{R}{\kappa^2} + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - U - J \phi \right) d^4x, \quad (5)$$

where $U = U(\phi)$ denotes the potential of the ϕ -field, $J = J(x^\mu)$ is its source term and we have set $\kappa = \sqrt{2\chi}$. Clearly, the argument of Landau and Lifshitz applies

⁴The sole quotation of Noerdlinger's paper we have found yet in the literature is from a paper of Nordtvedt [6] that goes back to 1970 (see also Ni [7]).

without inconsistency because the signs of the Ricci scalar, R , and of the kinetic term of the ϕ -field are opposit. Furthermore, this remains true for other kinds of covariant couplings (in particular for a conformal coupling) and for a complex scalar field.

The action for the classical five dimensional spacetime Kaluza-Klein theory reads (hereafter, any quantity carrying a hat is five dimensional, the other notations are obvious)

$$S_{KK} = - \int \sqrt{-\hat{g}} \frac{\hat{R}}{\hat{\kappa}^2} d^5x. \quad (6)$$

Relation (6) can be expressed, after dimensional reduction, either in the Jordan-Fierz frame or in the Einstein-Pauli frame. One passes from the point of view of the Jordan-Fierz frame to that of the Einstein-Pauli frame by the conformal transformation

$$\hat{g}_{AB} \rightarrow \tilde{\phi}^{-1/3} \hat{g}_{AB} \quad (7)$$

and the following redefinition of the ϕ -field

$$\hat{g}_{44} = \phi^2 \rightarrow \hat{g}_{44} = \tilde{\phi}, \quad (8)$$

where the \hat{g}_{AB} (resp. the \hat{g}^{AB}) denote the covariant (resp. contravariant) components of the five dimensional metric ; A, B (resp. M, N) = 0, 1, 2, 3, 4. One gets,

1. in the Jordan-Fierz frame [10], [11] :

$$S_{KK} = - \int \sqrt{-g} \phi \left(\frac{R}{\kappa^2} + \frac{1}{4} \phi^2 F^{\mu\nu} F_{\mu\nu} + \frac{2}{\kappa^2} \frac{\partial^\mu \phi \partial_\mu \phi}{\phi^2} \right) d^4x, \quad (9)$$

2. in the Einstein-Pauli frame [11], [12], [13], [14] :

$$S_{KK} = - \int \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{\kappa^2} + \frac{1}{4} \tilde{\phi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{6\kappa^2 \tilde{\phi}^2} \partial^\mu \tilde{\phi} \partial_\mu \tilde{\phi} \right) d^4x, \quad (10)$$

where $\kappa^{-2} = \int \hat{\kappa}^{-2} dx^4$ and the $F_{\mu\nu}$ (resp. $F^{\mu\nu}$) are the covariant (resp. contravariant) components of the electromagnetic strength tensor : $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (resp. $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$).

Relations (10) and (9) are equivalent only with respect to the ground state of the ϕ -field the vacuum expectation value of which is $\langle \phi \rangle = 1$.

As one can see (by comparing relations (10) and (5) or relations (9) and (5)), in both frames the sign of the kinetic part of the ϕ -field in the Kaluza-Klein lagrangian density is negative. Hence, unless one considers the limiting case $\phi = \text{constant}$ as did indeed Kaluza [15] and Klein [16], applying analogously the argument of Landau and Lifshitz to relation (10) or (9) reveals an inconsistency. Indeed, as first pointed out by Thiry [10] and Jordan [17], it is well known that the case $\phi = \text{constant}$ is too restrictive requiring the strict equality of the magnitudes of the magnetic field and the electric field (up to the factor c , velocity of light in the vacuum). Thus, we conclude that the classical Kaluza-Klein theory is unstable, in the sense that its action turns out to be unbounded from below.

seems to hold whatever the number of dimensions of the internal space may be (see the expression of the action obtained by Cho and Keum [18] for the four-dimensional Einstein-Yang-Mills theory after dimensional reduction). To our knowledge, this point has never been discussed hitherto. Perhaps this comes from the belief that, when one neglects the electromagnetic field, the Kaluza-Klein theory reduces to the special case $\omega = 0$ of the Brans-Dicke theory (which is true but needs further discussion). Also, errors on the sign of the kinetic term of the Kaluza-Klein scalar are frequent in the literature, different from one author to another, even using the same signature of the metric. In order to be confident on the latter point, we have carried out the calculations of the Kaluza-Klein action both in Jordan-Fierz and Einstein-Pauli frames (see appendix). In the Jordan-Fierz frame, we have found the same expression for the five dimensional Ricci scalar as Thiry [10] (see also Licherowicz [19]) who used the Cartan method and an orthonormal moving frame. Furthermore, let us notice that the inspection of the supergravity lagrangian den-

sity (see Bergshoeff *et al.* [20]) shows that the same remark and the same conclusion as for the Kaluza-Klein theory may be made for supergravity in ten dimensions.

3 Conclusion

We have shown the inconsistency of the purely geometrical Kaluza-Klein program, due to the sign of the kinetic term of the scalar field which leads to an action unbounded from below and thus to instabilities. Thus we claim that any theory which leads to this form, as low energy limit, must be rejected unless an efficient stabilizing mechanism is provided. In a forthcoming paper, we will propose such a solution, restoring a lower bound for the action in the framework of the Kaluza-Klein five dimensional unification theory. The perturbing negative sign in the Kaluza-Klein action is compensated by introducing an additional real (external) scalar field, which is minimally coupled to gravity. We do not claim that this is the only possibility but this would bring some more clarification on the central role of the Higgs field in particle physics or inflaton in cosmology.

4 Appendix : Calculation of the five dimensional Kaluza-Klein action

We adopt the signature + - - - . The five dimensional Ricci scalar reads

$$\hat{R} = \hat{g}^{AB} (\partial_N \hat{\Gamma}_{AB}^N - \partial_B \hat{\Gamma}_{AN}^N + \hat{\Gamma}_{AB}^N \hat{\Gamma}_{NM}^M - \hat{\Gamma}_{AN}^M \hat{\Gamma}_{BM}^N). \quad (11)$$

This involves (see Landau and Lifshitz [1]) :

$$\sqrt{-\hat{g}} \hat{R} = \sqrt{-\hat{g}} \hat{G} + \hat{D}, \quad (12)$$

with

$$\hat{G} = \hat{g}^{AB} (\hat{\Gamma}_{AN}^M \hat{\Gamma}_{BM}^N - \hat{\Gamma}_{AB}^N \hat{\Gamma}_{NM}^M) \quad (13)$$

and, on account of the cylinder condition,

$$\hat{D} = \partial_\nu (\sqrt{-\hat{g}} \hat{g}^{AB} \hat{\Gamma}_{AB}^\nu) - \partial_\beta (\sqrt{-\hat{g}} \hat{g}^{A\beta} \hat{\Gamma}_{AN}^N). \quad (14)$$

Analogously to the conditions (1) set by Landau and Lifshitz for the three-dimensional space in spacetime, we compute the quantities (13) and (14) assuming the choice of a set of spacetime coordinates that respects the following conditions

$$\hat{g}_{4\mu} = 0 \quad \text{and} \quad \det(g_{\mu\nu}) = \text{constant}. \quad (15)$$

This choice strongly simplifies the calculations and thus avoids many errors. In particular, it is straightforward that⁵ $\hat{g}^{4\mu} = 0$, $\hat{g}^{44} = 1/\hat{g}_{44}$, $\hat{g} = \det(\hat{g}_{AB}) = g \hat{g}_{44}$, $\hat{\Gamma}_{\alpha\beta}^\nu = \Gamma_{\alpha\beta}^\nu$, $\Gamma_{\alpha\beta}^\beta = 0$, $\sqrt{-\hat{g}} \partial_\beta (\hat{g}^{4\beta} \hat{\Gamma}_{4\nu}^\nu) = \partial_\beta (\sqrt{-\hat{g}} \hat{g}^{4\beta} \hat{\Gamma}_{4\nu}^\nu)$, $\sqrt{-\hat{g}} \partial_\nu (\hat{g}^{4\alpha} \hat{\Gamma}_{4\alpha}^\nu) = \partial_\nu (\sqrt{-\hat{g}} \hat{g}^{4\alpha} \hat{\Gamma}_{4\alpha}^\nu)$ and $(\partial_\nu \sqrt{-\hat{g}}) g^{\alpha\beta} \Gamma_{\alpha\beta}^\nu = 0$ (recall that for any scalar quantity, Ω , one has identically $\partial_\nu \Omega (\partial_\alpha g^{\nu\alpha}) = 0$). In addition, the cylinder condition implies $\hat{\Gamma}_{44}^4 = 0$. So, one is left with the following expressions

$$\hat{G} = G + 2g^{\alpha\beta} \hat{\Gamma}_{\alpha 4}^\mu \hat{\Gamma}_{\beta\mu}^4 + g^{\alpha\beta} \hat{\Gamma}_{\alpha 4}^4 \hat{\Gamma}_{\beta 4}^4 + \hat{g}^{44} \hat{\Gamma}_{4\nu}^\mu \hat{\Gamma}_{4\mu}^\nu + \hat{g}^{44} \hat{\Gamma}_{44}^\mu \hat{\Gamma}_{4\mu}^4 \quad (16)$$

and, dropping the total divergence terms $\partial_\beta (\sqrt{-\hat{g}} \hat{g}^{4\beta} \hat{\Gamma}_{4\nu}^\nu)$ and $\partial_\nu (\sqrt{-\hat{g}} \hat{g}^{4\alpha} \hat{\Gamma}_{4\alpha}^\nu)$,

$$\hat{D} = \sqrt{\hat{g}_{44}} D - \left[\frac{(\partial_\nu \sqrt{\hat{g}_{44}})}{\sqrt{\hat{g}_{44}}} \hat{g}^{44} \hat{\Gamma}_{44}^\nu - \partial_\nu (\hat{g}^{44} \hat{\Gamma}_{44}^\nu) + \frac{(\partial^\alpha \sqrt{\hat{g}_{44}})}{\sqrt{\hat{g}_{44}}} \hat{\Gamma}_{\alpha 4}^4 + \partial_\beta (g^{\alpha\beta} \hat{\Gamma}_{\alpha 4}^4) \right] \sqrt{-\hat{g}}, \quad (17)$$

where the quantities

$$G = g^{\alpha\beta} (\Gamma_{\alpha\nu}^\mu \Gamma_{\beta\mu}^\nu - \Gamma_{\alpha\beta}^\nu \Gamma_{\nu\mu}^\mu) \quad (18)$$

and

$$D = \partial_\nu (\sqrt{-g} g^{\alpha\beta} \Gamma_{\alpha\beta}^\nu) - \partial_\beta (\sqrt{-g} g^{\alpha\beta} \Gamma_{\alpha\nu}^\nu). \quad (19)$$

⁵Relation (15) does not involve the cancellation of the derivatives of the $\hat{g}_{4\mu}$'s unlike the case for the derivatives of $g = \det(g_{\mu\nu})$. This is well understood if one remembers that the $\hat{g}_{4\mu}$'s, in the gauge theories point of view, are both potentials and connections.

are respectively the spacetime analogous of \hat{G} and \hat{D} . Similarly, the spacetime analogous of relation (12) reads

$$\sqrt{-g} R = \sqrt{-g} G + D. \quad (20)$$

As one can see whereas D is a total divergence of spacetime, this is not the case for \hat{D} since the terms involving the partial derivatives with respect to the fifth coordinate are missing because of the cylinder condition. At this point, the only Christoffel symbols we have to compute explicitly are these of the form $\hat{\Gamma}_{\alpha 4}^4$, $\hat{\Gamma}_{44}^\mu$, $\hat{\Gamma}_{4\nu}^\mu$ and $\hat{\Gamma}_{\beta\mu}^4$. One gets the well known relations

$$\hat{\Gamma}_{\alpha 4}^4 = \frac{1}{2} \hat{g}^{44} \partial_\alpha \hat{g}_{44}, \quad (21)$$

$$\hat{\Gamma}_{44}^\alpha = -\frac{1}{2} \partial^\alpha \hat{g}_{44}, \quad (22)$$

$$\hat{\Gamma}_{4\nu}^\mu = \frac{1}{2} g^{\mu\alpha} (\partial_\nu \hat{g}_{4\alpha} - \partial_\alpha \hat{g}_{4\nu}) \quad (23)$$

and

$$\hat{\Gamma}_{\beta\mu}^4 = \frac{1}{2} \hat{g}^{44} (\partial_\mu \hat{g}_{\beta 4} + \partial_\beta \hat{g}_{4\mu}). \quad (24)$$

Hence, it comes

$$\hat{\Gamma}_{4\nu}^\nu = 0 \quad (25)$$

and

$$\partial_\nu (\hat{g}^{44} \hat{\Gamma}_{44}^\nu) = -\frac{1}{2} (\partial_\nu \hat{g}^{44} \partial^\nu \hat{g}_{44} + \hat{g}^{44} \partial_\nu \partial^\nu \hat{g}_{44}). \quad (26)$$

Moreover, one checks easily that

$$g^{\alpha\beta} \hat{\Gamma}_{\alpha 4}^\mu \hat{\Gamma}_{\beta\mu}^4 = 0 \quad (27)$$

and

$$g^{\alpha\beta} \hat{\Gamma}_{\alpha 4}^4 \hat{\Gamma}_{\beta 4}^4 + \hat{g}^{44} \hat{\Gamma}_{44}^\mu \hat{\Gamma}_{4\mu}^4 = 0. \quad (28)$$

Thus relations (16) and (17) reduce respectively to

$$\hat{G} = G + \frac{1}{4} \hat{g}^{44} g^{\mu\alpha} g^{\nu\beta} (\partial_\nu \hat{g}_{4\alpha} - \partial_\alpha \hat{g}_{4\nu}) (\partial_\mu \hat{g}_{4\beta} - \partial_\beta \hat{g}_{4\mu}) \quad (29)$$

and

$$\hat{D} = \sqrt{\hat{g}_{44}} D - [\partial_\nu (\hat{g}^{44} \partial^\nu \hat{g}_{44})] \sqrt{-\hat{g}}. \quad (30)$$

Hence, replacing relations (29) and (30) in relation (12) yields on account of relation (20) :

$$\hat{R} = R + \frac{1}{4} \hat{g}^{44} g^{\mu\alpha} g^{\nu\beta} (\partial_\nu \hat{g}_{4\alpha} - \partial_\alpha \hat{g}_{4\nu}) (\partial_\mu \hat{g}_{4\beta} - \partial_\beta \hat{g}_{4\mu}) - \partial_\nu (\hat{g}^{44} \partial^\nu \hat{g}_{44}). \quad (31)$$

Now, the above relation is equivalent to the following

$$\hat{R} = R + \frac{1}{4 \hat{g}_{44}} g^{\mu\alpha} g^{\nu\beta} (\partial_\nu \hat{g}_{4\alpha} - \partial_\alpha \hat{g}_{4\nu}) (\partial_\mu \hat{g}_{4\beta} - \partial_\beta \hat{g}_{4\mu}) + \frac{1}{2} \frac{\partial_\nu \hat{g}_{44} \partial^\nu \hat{g}_{44}}{(\hat{g}_{44})^2} \quad (32)$$

since

$$\begin{aligned} \sqrt{-\hat{g}} \partial_\nu (\hat{g}^{44} \partial^\nu \hat{g}_{44}) &= \partial_\nu (\sqrt{-\hat{g}} \hat{g}^{44} \partial^\nu \hat{g}_{44}) - \hat{g}^{44} (\partial^\nu \hat{g}_{44}) (\partial_\nu \sqrt{-\hat{g}}) \\ &= \partial_\nu (\sqrt{-\hat{g}} \hat{g}^{44} \partial^\nu \hat{g}_{44}) - \frac{1}{2} \frac{\partial_\nu \hat{g}_{44} \partial^\nu \hat{g}_{44}}{(\hat{g}_{44})^2} \sqrt{-\hat{g}} \end{aligned}$$

and we may drop the total divergence for our concern. Clearly, the sign accompanying the kinetic term as regards the fifteen degree of freedom (identified to the scalar field, up to a power-law), \hat{g}_{44} , of the Kaluza-Klein theory is unambiguously negative.

In Jordan-Fierz frame, the fields potentials are defined by : $\hat{g}_{44} = \phi^2$, $\hat{g}_{4\mu} = \kappa \phi^2 A_\mu$ and $\hat{g}_{\mu\nu} = g_{\mu\nu} + \kappa^2 \phi^2 A_\mu A_\nu$. Carrying these relations into relation (32) yields expression (9) of the five dimensional Einstein-Hilbert action. In Einstein-Pauli frame, the potentials of the fields are defined by : $\hat{g}_{44} = \tilde{\phi}^{2/3}$, $\hat{g}_{4\mu} = \kappa \tilde{\phi}^{2/3} A_\mu$ and $\hat{g}_{\mu\nu} = \tilde{\phi}^{-1/3} (\tilde{g}_{\mu\nu} + \kappa^2 \tilde{\phi} A_\mu A_\nu)$. Carrying these relations into relation (32) yields expression (10) of the five dimensional Einstein-Hilbert action.

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